Football Sport League Scheduling Technique using the Polygon Construction Method

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Abstract

In the real world, the football sport league scheduling is very popular, while the people all over the world are encouraging and watching football matches interestingly. The existing situation according to the program of matches or schedule makes the football fans very interesting. Sports league scheduling is very important for both professional and amateur sports alike. Obtaining good "playable" schedules under a myriad of league and logistics constraints is extremely difficult yet essential to the success of the league. Like most team sports, the location (home and away) at which the game is played can significantly affect the outcome of the game. The home team has advantage because they are most familiar with the venue and the away team has a disadvantage, they must unfamiliarity with the venue. So balancing the number of games a team plays at home and away is important to ensure fairness. In this paper, we have clearly explained about the polygon construction method to generate the fair schedule.

1. Introduction

Scheduling problems in sports has become an important class of optimization problems in recent years [2, 5]. The professional sport leagues represent a major economic activity around the world.

For several sports, e.g. soccer, basketball, football, baseball, hockey, etc, where the teams are plays a double round-robin tournament among themselves, where the games are played in different places during some time period [4].

Thus schedules can be produced which include as many desirable properties as possible while still being feasible (playable) because to generate good scheduling is a very hard task.

2. Polygon Construction Method

The basic facts of polygon construction method are the even number of teams take part in a tournament [1, 2, 3, 6], each team has its own stadium at its home city, each team plays every other team exactly twice in **2(*n*-1)** rounds (once at home and once away), and double round-robin (**DRR**) is a single round-robin (**SRR**) tournament in the first (***n*-1**) rounds, followed by the same **SRR** tournament with reversed venues in the last (***n*-1**) rounds [5, 6].

2.1 Game Assignment

Step 1: ***User input***

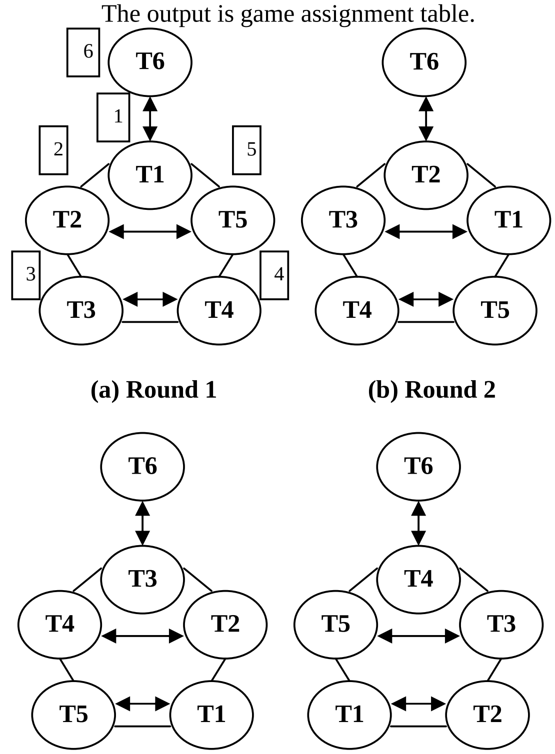
The initial user input is total number of teams (***n***, even number) [1, 6] and the name of teams.

Step 2: ***Process***

The team represents the position **1, 2 … *n***. The teams of position **i= 1, 2 … *n*/2** plays with the teams of positions ***n*-i+1**. The team of position **1** moved to the position ***n*-1** and the teams of positions **2, 3 … *n*-1** are move to the previous positions for all rounds, the positions are anti-clockwise form, (see Figure 1.).

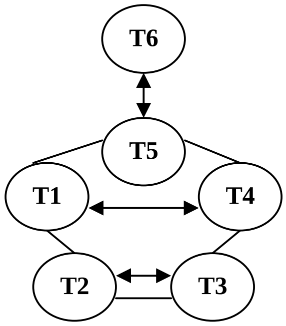
Step 3: ***Output***

The output is game assignment table.



(a) Round 1 **(b) Round 2**

(c) Round 3 **(d) Round 4**



(e) Round 5

Figure 1. Game assignment

2.2 Stadium assignment

Step 1: ***Input***

The input is game assignment table. In this section, we assign the stadium.

Step 2: ***Process***

We define the last position team is base team. The base team is home and away and the position **1** team is away and home alternately in round **1** to ***n*-1**. The teams of odd positions are home and the teams of even positions are away in odd rounds (**1, 3 … *n*-1**), and the teams of even positions are home and the teams of odd positions are away in even rounds (**2, 4 … *n*-2**), see also Figure 1.

Step 3: ***Output***

The outputs are temporary home-away tables ($T^k = t^k_{i,r}$).

In this section,
- $T^k = t^k_{i,r}$ represent temporary home-away table.
- ***k*** represent base team (last position team).
- ***i*** represent a position pointing a team.
- ***r*** represent rounds.

2.3 Break and equity

2.3.1 Break

In this section, we see the following theorems are a well-known facts on sport scheduling-

Theorem 1: In any timetable of ***n*** teams, the number of breaks is greater than or equal to ***n*-2** [6].

Theorem 2: If a timetable of ***n*** teams has ***n*-2** breaks, then exactly two teams have no break and others have exactly one break [6].

Step 1: ***Input***

The inputs are stadium assignment tables.

Step 2: ***Process***

If a team plays two games either both at their home or both at away in slot ***s*** and ***s*+1**, we say that the team has a break at slot ***s*+1**. So, we count the number of breaks.

Step 3: ***Output***

The output is total break.

2.3.2 Equity

Step 1: ***Input***

The data are stadium assignment tables.

Step 2: ***Process***

We calculate total equity score because a schedule should equitable and balance [6].

$$Equity = \sum_{t \epsilon N} max\,(0, H_t - \frac{n}{2})$$

Summing for all teams, equity is optimal (fair) when resulting in a final equity score of zero.

Step 3: ***Output***

The outputs are total equity scores.

In this section,
- ***N*** represents set of all teams.
- ***t*** represent team in symbol ***N***.
- H_t represents the number of games played at home by team ***t***.

2.4 True-False assignment ($x_{i,r}$)

Step 1: ***Input***

The input is temporary home-away tables (all stadium tables) because we reduce the number of consecutive home or away games.

Step 2: ***Process***

A true-false assignment $x \in \{TRUE, FALSE\}$ $((i,r) \in N\setminus\{k\} \times S)$, satisfying the conditions that-

(a) Main diagonal:

$x_{i,s_{ki}} = TRUE$ $(i \in N\setminus\{k\})$

(b) Upper bound:

(i) $x_{i,s_{ij}} \neq x_{j,s_{ij}}$ $(\{i,j\} \subseteq N\setminus\{k\},\ t^k_{i,s_{ij}} = t^k_{j,s_{ij}})$

(ii) $x_{i,s_{ij}} = x_{j,s_{ij}}$ $(\{i,j\} \subseteq N\setminus\{k\},\ t^k_{i,s_{ij}} \neq t^k_{j,s_{ij}})$

Where $i = 1, 2 \dots n-2$ and $j = i+1$.

(iii) $(x_{i,s_{ki}} \vee \neg\, x_{i,s_{ki}+1}) = (x_{i,s_{ki}+1} \vee \neg\, x_{i,s_{ki}+2}) = \dots$

$= \dots = (x_{i,2n-1} \vee \neg\, x_{i,1}) = (x_{i,1} \vee \neg\, x_{i,2}) = \dots$

$= \dots = (x_{i,s_{ki}-2} \vee \neg\, x_{i,s_{ki}-1}) = TRUE$ $(i \in N\setminus\{k\})$

If it exists, else ***FALSE***.

(c) Lower bound:

$s_l = s_u$

Where $l = 1, 2 \dots n-2$ and $u = n-1, n-2 \dots 2$.

Step 3: ***Output***

The output is true-false assignment results.

In this section, we used the following symbols. These are
- $x_{i,r}$ represent true-false assignment format.
- ***k*** represent base team.
- ***S*** represents set of slots (rounds).
- ***i, j*** represents a position pointing a team.
- ***l, u*** represents a position positing a team.

2.5 Home-Away table ($T' = t'_{i,r}$)

Step 1: ***Input***

The input data are temporary home-away table (all stadium tables) and true-false assignment results.

Step 2: ***Process***

We compare two tables, if true-false assignment is ***TRUE***, the same as temporary home-away table; else we change home to away and away to home. The conditions that-

$$t'_{i,r} = \begin{cases} t^k_{i,r}\ (x'_{i,r} = TRUE) \\ A\ (x'_{i,r} = FALSE \text{ and } t'_{i,r} = H) \\ H\ (x'_{i,r} = FALSE \text{ and } t'_{i,r} = A) \end{cases}$$

Step 3: ***Output***

The outputs are home-away schedules.

In this section,

- $T' = t'_{i,r}$ represent home-away table format.
- ***H*** and ***A*** represents home and away.

2.6 Recalculating break and equity

Step 1: ***Input***

The input data are home-away tables.

Step 2: ***Process***

We calculate total breaks and total equity scores, also section 2.3.1 and section 2.3.2.

Step 3: ***Output***

The output is total breaks and total equity scores for all home-away tables.

2.7 Generating the fair scheduling

Step 1: ***Input***

The input data are home-away tables, total breaks and total equity scores.

Step 2: ***Process***

If break is ***n*-2** and equity is zero, we reverse that home-away table's venues for second-half because a double round-robin [4] is a tournament where each team plays every other once in the ***n*-1** rounds, followed by the same games with reversed venues in the last ***n*-1** rounds.

Step 3: ***Output***

The final output is fair schedule.

3. System processing and design

Initially, we received the total number of teams and the name of teams from user. And then we assigned the game and stadium. After that we produced temporary home-away tables (one was theory table and others were manual tables). We calculated total breaks and total equity scores for which tables. If the results were not equivalent to the theorems, we used the true false assignment formulas. And then we produced the home away tables. After that, we recalculated the total breaks and total equity scores. If the results of a table were equivalent to the theorems in home-away tables, final produced the fair schedule (that table and reversed). The system design is shown at Figure 2.

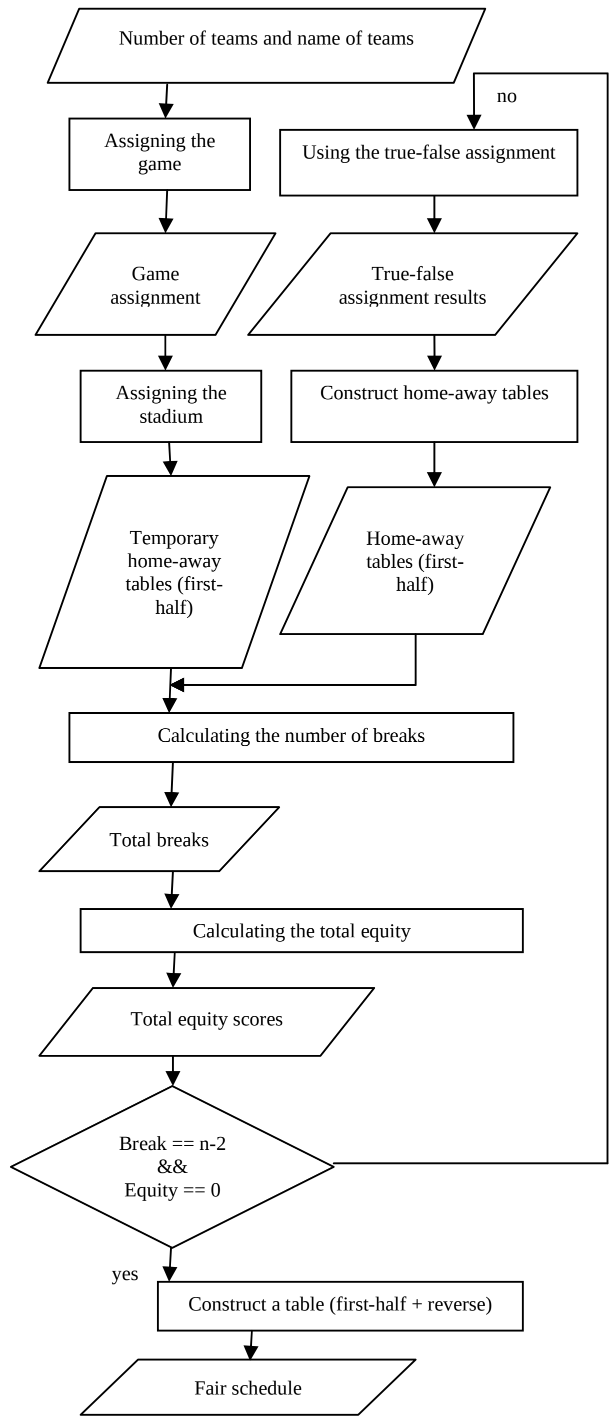


Figure 2. System design

4. Experimental results

4.1 Game assignment

We have analyzed the total six (***n***) teams to generate fair schedule and used team names T1, T2, T3, T4, T5 and T6. In this section, we assigned the game each two teams according to the section 2.1, shown in Table 1.

Table 1. Game assignment table

Team	Slots (Rounds)				
	1	2	3	4	5
T1	T6	T3	T5	T2	T4
T2	T5	T6	T4	T1	T3
T3	T4	T1	T6	T5	T2
T4	T3	T5	T2	T6	T1
T5	T2	T4	T1	T3	T6
T6	T1	T2	T3	T4	T5

4.2 Stadium assignment

In this section, we generated four stadium tables (see Table 2) for generate fair schedule, one is theory table and others are manual tables because we wanted to comparison our theory schedule and other manual schedules.

In Table 2, '@' means that the game is held at the home of the opponent (away), while without '@' means that the game is held at the home of the team corresponding to the row.

4.3 Break and equity

After that we checked four stadium tables according to Theorem 1 and Theorem 2 and then we also analyzed the total equity according to the processes in section 2.3.The total breaks and total equity score for all schedules are shown in the Table 3.

In this step, we could not generated fair schedule because all stadium tables' breaks are not equal to ***n*-2** (by Theorem 1), and all stadium tables' equity are not equal to zero. So, we used the true-false assignment formulas to generate the fair schedule.

4.4 Home-Away table using the true-false assignment formulas

In Table 3, all scheduling was not proved the theorems. So we generated four home-away tables using true-false assignment formulas according to the section 2.4 and 2.5. These are shown in Table 4.

Table 2. Stadium(S) tables

S table	Team	Slots (Rounds)				
		1	2	3	4	5
1 (manual table 1)	T1	@T6	T3	@T5	T2	@T4
	T2	@T5	@T6	T4	@T1	T3
	T3	T4	@T1	@T6	T5	@T2
	T4	@T3	T5	@T2	@T6	T1
	T5	T2	@T4	T1	@T3	@T6
	T6	T1	T2	T3	T4	T5
2 (manual table 2)	T1	T6	@T3	T5	@T2	T4
	T2	T5	T6	@T4	T1	@T3
	T3	@T4	T1	T6	@T5	T2
	T4	T3	@T5	T2	T6	@T1
	T5	@T2	T4	@T1	T3	T6
	T6	@T1	@T2	@T3	@T4	@T5
3 (manual table 3)	T1	T6	@T3	@T5	T2	T4
	T2	T5	T6	@T4	@T1	T3
	T3	T4	T1	T6	@T5	@T2
	T4	@T3	@T5	T2	T6	@T1
	T5	@T2	T4	T1	T3	T6
	T6	@T1	@T2	@T3	@T4	@T5
4 (theory table)	T1	@T6	@T3	@T5	@T2	@T4
	T2	@T5	T6	T4	T1	T3
	T3	T4	T1	@T6	@T5	@T2
	T4	@T3	@T5	@T2	T6	T1
	T5	T2	T4	T1	T3	@T6
	T6	T1	@T2	T3	@T4	T5

Table 3. Break and equity for all stadium tables

Stadium table	Total break	Total equity
1	8	2
2	8	0
3	16	0
4	16	2

4.5 Recalculating the break and equity

After that we also checked four home-away schedules according to Theorem 1 and Theorem 2 and then we also analyzed the total equity according to the processes in section 2.3 (also section 4.3). The total breaks and total equity scores for all schedules are shown in the Table 5.

4.6 Generating fair schedule

In all home-away schedules, schedule number 4 (our theory table) was equivalence to the Theorem 1 and Theorem 2, total break was equal to ***n*-2** and total equity score was equal to zero. So, we generated the fair schedule with that schedule and reversed the venues for second-half, shown in Table 6.

Table 4. Home-Away(HA) tables

HA table	Team	Slots (Rounds)				
		1	2	3	4	5
1 (manual table 1)	T1	@T6	@T3	@T5	@T2	@T4
	T2	@T5	@T6	T4	T1	T3
	T3	T4	T1	@T6	@T5	@T2
	T4	@T3	@T5	@T2	@T6	T1
	T5	T2	T4	T1	T3	@T6
	T6	T1	T2	T3	T4	T5
2 (manual table 2)	T1	T6	T3	T5	T2	T4
	T2	T5	T6	@T4	@T1	@T3
	T3	@T4	@T1	T6	T5	T2
	T4	T3	T5	T2	T6	@T1
	T5	@T2	@T4	@T1	@T3	T6
	T6	@T1	@T2	@T3	@T4	@T5
3 (manual table 3)	T1	T6	T3	@T5	@T2	T4
	T2	T5	T6	@T4	T1	T3
	T3	T4	@T1	T6	T5	@T2
	T4	@T3	@T5	T2	T6	@T1
	T5	@T2	T4	T1	@T3	T6
	T6	@T1	@T2	@T3	@T4	@T5
4 (theory table)	T1	@T6	T3	@T5	T2	@T4
	T2	@T5	T6	T4	@T1	T3
	T3	T4	@T1	@T6	T5	@T2
	T4	@T3	T5	@T2	T6	T1
	T5	T2	@T4	T1	@T3	@T6
	T6	T1	@T2	T3	@T4	T5

Table 5. Break and equity for all home-away schedules

Schedule	Total breaks	Total equity
1	20	3
2	20	3
3	12	1
4	4	0

Table 6. Fair schedule

First-half (Home Vs Away)			
Round	**Match 1**	**Match 2**	**Match3**
1	T6 Vs T1	T5 Vs T2	T3 Vs T4
2	T2 Vs T6	T1 Vs T3	T4 Vs T5
3	T6 Vs T3	T2 Vs T4	T5 Vs T1
4	T4 Vs T6	T3 Vs T5	T1 Vs T2
5	T6 Vs T5	T4 Vs T1	T2 Vs T3
Second-half (Home Vs Away)			
Round	**Match 1**	**Match 2**	**Match 3**
6	T1 Vs T6	T2 Vs T5	T4 Vs T3
7	T6 Vs T2	T3 Vs T1	T5 Vs T4
8	T3 Vs T6	T4 Vs T2	T1 Vs T5
9	T6 Vs T4	T5 Vs T3	T2 Vs T1
10	T5 Vs T6	T1 Vs T4	T3 Vs T2

5. Conclusion

Sports league schedule has two main limitations. First, any team should not more than three consecutive home or away games. Second, a game of team ***i*** at team ***j***'s home cannot be followed by the game of team ***j*** at team ***i***'s home. If we review our schedule, any teams have not more than three consecutive home or away games. If we see the total breaks, the total break number is ***n*-2**. And then, the equity is equal to zero (optimal). Therefore, our fair schedule is equivalence to the limitations, theorems and our discovering points.

6. References

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